TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 4 2022

Section 1 Calculator Free
Logarithms

STUDENT'S NAME ______________________________

DATE: Thursday 30th June **TIME:** 25 minutes **MARKS:** 29

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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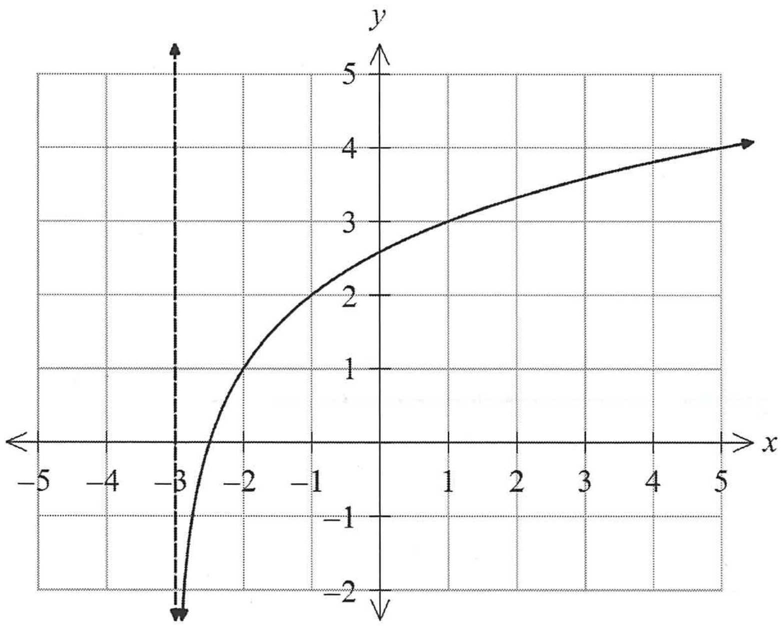
1. (3 marks)

Consider the function $y = f(x)$ graphed below.



Given $f(x) = \log_a(x - b) + c$, determine the values of a, b, and c.

$a = 2$ ✓ $b = 3$ ✓ $c = 1$ ✓

✓ one correct ✓✓ two correct ✓✓✓ three correct.

2. (7 marks)

Differentiate each of the following with respect to x.

(a) $\ln(x^3+4x-5)$ [1]

$\frac{d}{dx} = \frac{2x^2+4}{x^3+4x-5}$ ✓

✓ differentiates correctly.

(b) $e^{2x}\ln(\sqrt{x}+5)$ [2]

$2e^{2x}.\ln(\sqrt{x}+5). + \frac{e^{2x}.\frac{1}{2}x^{-1/2}}{\sqrt{x}+5}$

✓ ✓

✓ differentiates $\ln(\sqrt{x}+5)$

✓ uses product rule for correct ans.

(c) $\ln\left(\frac{x-6}{(3x+5)^4}\right)$ [4]

$\ln(x-6) - \ln(3x+5)^4$ ✓

$= \ln(x-6) - 4.\ln(3x+5)$ ✓

$\frac{d}{dx} = \frac{1}{x-6} - \frac{12}{3x+5}$

✓ ✓

$\ln(x-6) - \ln(3x+5)^4$ ✓

$\frac{d}{dx} = \frac{1}{x-6} - \frac{4(3x+5)^3}{(3x+5)^4}$ ✓

✓ ✓

✓ splits log.

✓ bring power down.

✓ differentiates one part

✓ differentiates both parts

✓splits log.

✓ differentiates first part

✓✓ uses chain rule to differe. second part.

3. (10 marks)

(a) Express the expression $2\log(4x+3)$ in terms of the natural logarithm. [2]

✓ splits to natural log. with c.o.B.

✓ correct final ans.

$$\frac{2\ln(4x+3)}{\ln 10} \checkmark\checkmark \quad \underline{\text{or}} \quad \frac{\ln[(4x+3)^2]}{\ln 10} \checkmark\checkmark$$

(b) Solve exactly for x in each of the following equations.

(i) $\log_x 5 = 0.5$ [1]

✓ correct ans

$x = 25$ ✓

(ii) $\log_4 x - \log_4(x+3) = -1$ [3]

✓ writes as single log.

✓ converts to expenential

✓ solves for x.

$\log_4\left(\frac{x}{x+3}\right) = -1$ ✓

$4^{-1} = \frac{x}{x+3}$ ✓

$x - 3 = 4x$

$3x = 3$

$\therefore x = 1$ ✓

(iii) $6^{x-1} = 2^{x+1}$ [4]

✓ takes logs of both sides

✓ brings power to front and expands

✓ takes out factor of x.

✓ solves for x.

$\log 6^{x-1} = \log 2^{x+1}$ ✓

$(x-1)\log 6 = (x+1)\log 2$ ✓

$x.\log 6 - \log 6 = x\log 2 + \log 2.$

$x(\log 6 - \log 2) = \log 2 + \log 6$ ✓

$x\log 3 = \log 12$

$x = \frac{\log 12}{\log 3}$ ✓

4. (9 marks)

(a) Evaluate $\int_1^2 \left(e^x + \frac{1}{x}\right) dx$. [3]

$= \left[e^x + \ln(x)\right]_1^2$ ✓

$= \left[e^2 + \ln(2)\right] - \left[e^1 - \ln(1)\right]$ ✓

$= e^2 - e + \ln 2$ ✓

✓ antidifferentiates correctly.

✓ subs in correctly.

✓ evaluates final ans.

(b) If $\frac{dV}{dt} = \frac{(2t-1)(2t+1)}{t}$, determine V in terms of t, given that the function V passes through the point $(1,5)$. [4]

✓ expand and split dV/dt

✓ antidiff for V

✓ sub in (1,5)

✓ states final equation.

$\frac{dV}{dt} = \frac{4t^2-1}{t}$

$= 4t - \frac{1}{t}$ ✓

$V = 2t^2 - \ln t + c$ ✓

$5 = 2(1)^2 - \ln(1) + c$ ✓

$5 = 2 + c$

$c = 3$

$\therefore V = 2t^2 - \ln t + 3$ ✓

(c) Determine the antiderivative of $\frac{\pi}{\tan x}$ with respect to x. [2]

$\int \frac{\pi \cos x}{\sin x} dx$ ✓

✓ writes $\frac{\pi}{\tan x}$ as $\frac{\pi \cos x}{\sin x}$.

✓ antidifferentiates correctly

$f(x) = \sin x$

$f'(x) = \cos x$.

$\therefore = \pi \ln(\sin x) + c$ ✓

TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 4 2022

Section 2 Calculator Assumed

Logarithms

STUDENT'S NAME ______________________________

DATE: Thursday 30th June **TIME:** 20 minutes **MARKS:** 20

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (3 marks)

pH is a measure of how acidic or alkaline a substance is, and the pH scale goes from 0 to 14, 0 being most acidic and 14 being most alkaline. Water in a stream has a neutral pH of about 7. The pH (p) of a substance can be found according to the formula $p = -\log h$ where h is the substances hydrogen ion concentration.

(a) A bottle of apple juice purchased has a hydrogen ion concentration of about $h = 0.0002$. Determine the pH of the apple juice, correct to one decimal place and hence state whether it is acidic or alkaline. [2]

✓ determines pH.

$p = -\log(0.0002)$ $\therefore$ acidic

✓ interprets acidic

$= 3.7$ ✓ ✓

(b) A banana has a pH of about 8.3. Determine the concentration of hydrogen ions, leaving your answer as an exact value. [1]

✓ writes as exponent and leaves ans as $10^{-8.3}$.

$8.3 = -\log h$

$h = 10^{-8.3}$ ✓

6. (8 marks)

An oil tanker is leaking at the rate $L'(t) = \dfrac{80\ln(t+1)}{t+1}$, where $L'(t)$ is hundreds of litres per hour and t is the number of hours after the leak occurs.

(a) Determine the initial rate of the leak. [1]

$L'(0) = \dfrac{80\ln(0+1)}{0+1} = 0$ ✓

✓ correct ans.

(b) Determine the total volume of oil that the ship will leak on:

(i) the first day. [2]

$\int_0^{24} \dfrac{80\ln(t+1)}{t+1}$ ✓ $= 414446\ L$ ✓

$= 414.446$

✓ determines def in between $0 \to 24$.

✓ multiply by $\times 100$

(ii) the second day. [1]

$\int_{24}^{48} \dfrac{80\ln(t+1)}{t+1} = 191404\ L$

$= 191.404$ ✓

(only −1 for units) $\times 100$.

✓ correct ans.

(c) Comment on the rate of the oil leak as t increases. [1]

it decreases. ✓

✓ decreases.

(d) The leak is repaired after the oil tanker has spilled 150 kL of oil into the ocean. Determine how many days after the initial leak the oil tanker is repaired. [3]

✓ writes statement with antideriv between $\int_0^k = 1500$

✓ determines antiderivative.

✓ states soln. 19 days

$150\,000\ L$

$1500 = \int_0^k \dfrac{80\ln(t+1)}{(t+1)}\,dt$ ✓

$1500 = 40(\ln(k+1)^2)$ ✓

$k = 455.56$

∴ on the 19th day. ✓

7. (9 marks)

An object has a displacement function $s(t) = t - \ln(8t + 1)$ where s is in metres and t is in seconds. Determine:

(a) the initial position of the object. [1]

$s(0) = 0 - \ln(8(0) + 1)$

$= 0$ m ✓

✓ initial position $= 0$

(b) the velocity function of the object. [1]

$s'(t) = \dfrac{8t - 7}{8t + 1}$ or $s'(t) = 1 - \dfrac{8}{8t+1}$ ✓

✓ differentiates correctly.

(c) at what time the object changes direction. [2]

$s'(t) = 0$ ✓

$\therefore t = 0.875$ s. ✓

✓ equates derivative to 0.

✓ solves for $t = 0.875$

(d) how far the object travels in the first 5 seconds. [3]

✓ determines $s(0.875)$

✓ determines $s(5)$

✓ calculates distance.

$t = 0.875 \quad = 0.875 - \ln(8(0.875) + 1)$.

$= -1.204$. ✓

$t = 5 \quad = 5 - \ln(8(5) + 1)$

$= 1.286$ ✓

$\therefore$

$1.204 + (1.286 - (-1.204$

$= 3.70$ m. ✓

(e) at what time the object returns to the origin. [2]

$0 = t - \ln(8t + 1)$ ✓

$t = 3.31$ s. ✓

✓ equates displacement function to 0.

✓ solves for $t = 3.31$ s.